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ON THE KINETIC THEORY OF THERMAL HYDRODYNAMIC FLUCTUATIONS IN INHOMOGENEOUS GAS*

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Equations are derived for thermal fluctuations of hydrodynamic fields in inhomogeneous gas streams. A modification of the Chapman—Enskog method of deriving standard solutions of the Boltzmann—Langevin kinetic equation for unbalanced phase density fluctuation is used. Expressions independent of thermodynamic mean fluxes are obtained for correlators of external heat lux sources and stress tensor fluctuations. Obtained formulas generalize the Landau—Litshitz formulas and extend the fluctuation-dissipation theorem to the domain of nonequilibrium stable states.

Recent interest in the theory of thermal noise in hydrodynamic systems is due to the possibility of extending it to the domain of nonequilibrium states. This is particularly important in the derivation of solutions of problems of simulation of unbalanced system anomalous behavior near the equilibrium threshold.

In the formal application of the equilibrium theory in investigations of unbalanced fluctuations one is confronted with the problem of determining the simultaneous statistical characteristics of hydrodynamic fields in the Onsager method, or of determining the statistical properties of external fluctuation sources in the Langevin method. Both problems are in essence equivalent to the problem of extending the fluctuation-dissipation theorem /1/ to the domain of unbalanced states. They have a complete solution /2—4/ at the kinematic level of the gas system evolution definition, and yield the Boltzmann—Langevin equation for the unbalanced phase density fluctuation /3/.

Basic equations of the kinetic theory /2—4/ represent a reasonable basis for subsequently passing to the hydrodynamic level of definition of transport processes and nonequilibrium thermal fluctuations in gas. The results obtained earlier in this way /5,6/ are substantially constrained by the condition of local thermodynamic equilibrium, and, consequently, do not take into account the effect of the system inhomogeneity on the statistical properties of external fluctuation sources in hydrodynamic equations representing a trivial extension of the equilibrium theory /7/. In investigations of a number of phenomena, in particular of the anomalous fluctuation increase in the region of stability threshold and their part in the process of turbulence onset /5,8,9/, it is necessary to take into account the effect of inhomogeneity on the statistical structure of fluctuating hydrodynamic fields. Investigation of these effects in gas streams is the subject of this paper.

1. Statistical structure of external fluctuation sources in equations of gasdynamics. In describing the kinetic development stage of the classical monatomic gas with allowance for large scale fluctuations we use the concept of random field of macroscopic densities of the state of system $N(t, x)$ in the μ-space whose mean value defines the single-particle distribution function $F(t, x) = \langle N(t, x)\rangle$, where $x = (\mathbf{r}, \mathbf{v})$ normalized with respect to the particles number N° /2—4/. Generally $N(t, x)$ satisfies the fairly complex nonlinear stochastic equation /4/. However in the domain of nonequilibrium but stable state of gas, in which the level of thermal fluctuation intensity $\delta N = N - F$ is low, that equation is considerably simplified /10/, splitting into the system of equations of the kinematic fluctuation theory /3/

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right) F(t, x) = J_v(F, F) \tag{1.1}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right) \delta N(t, x) = J_v'(F)\, \delta N(t, x) + \delta I(t, x) \tag{1.2}$$

where $J_v(F, F)$ and $J_v'(F)$ are Boltzmann integrals and the linearized collision operator, and $\delta I(t, x)$ is the Gaussian random field with zero mean value and the correlation function

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$$\langle \delta I(t_1, x_1)\, \delta I(t_2, x_2)\rangle = \delta(t_1 - t_2)\, \delta(\mathbf{r}_1 - \mathbf{r}_2)\, D[F, F;\ \mathbf{v}_1, \mathbf{v}_2] \equiv \delta(t_1 - t_2)\, \delta(\mathbf{r}_1 - \mathbf{r}_2)\, \{J_{v_1 v_2}(F, F) + \delta(\mathbf{v}_1 - \mathbf{v}_2)\, J_{v_1}(F, F) - [J_{v_1}'(F) + J_{v_2}'(F)]\, F(t_1, x_1)\, \delta(\mathbf{v}_1 - \mathbf{v}_2)\} \tag{1.3}$$

where $J_{v_1 v_2}(F, F)$ is the "nonintegrated collison integral" whose definition is given by formula (23.12) in /3/. Equations /1.1/ and /1.2/ with formula (1.3) constitute the mathematical basis of the considered here kinetic theory of hydrodynamic fluctuations in inhomogeneous gas streams.

The fluctuating collision integral δI in (1.2) is independent of δN and is, thus, an "external" fluctuation source in the Boltzmann equation linearized with respect to small deflections of phase density δN from its mean value F. By virtue of (1.3) its statistical properties depend on the extent of the gas nonequilibrium. It is important to point out the part played by individual terms in expression (1.3) in the formation of statistical structure of the random field δN. Using Eq.(1.1) we can represent the last three terms in the right-hand side of (1.3) as

$$\delta(t_1 - t_2)\left(\frac{\partial}{\partial t} + \sum_{i=1,2} [\mathbf{v}_i \cdot \nabla_i - J_{v_i}'(F)]\right) \delta(x_1 - x_2)\, F(t_1, x_1) \tag{1.4}$$

Taking this into account we obtain for the simultaneous fluctuation correlator the representation

$$\langle \delta N(t, x_1)\, \delta N(t, x_2)\rangle = \delta(x_1 - x_2)\, F(t, x_1) + g(t, x_1, x_2) \tag{1.5}$$

and for the three-dimensional correlation function g the equation

$$\left(\frac{\partial}{\partial t} + \sum_{i=1,2} [\mathbf{v}_i \cdot \nabla_i - J_{v_i}'(F)]\right) g(t, x_1, x_2) = \delta(\mathbf{r}_1 - \mathbf{r}_2)\, J_{v_1 v_2}(F, F) \tag{1.6}$$

Thus in (1.3) the effects related to statistical links between nonequilibrium gas volumes distributed in space are associated with the term $J_{v_1 v_2}$. In the state of local thermodynamic equilibrium $J_{v_1 v_2} = 0$ and Eq.(1.6) has a trivial solution which ensures the three-dimensional space δ-correlation of simultaneous fluctuations, related to the last terms in (1.3). At small deviations from the local thermodynamic equilibrium formula (1.3) ensures the appearance of nonequilibrium additions to the δ-correlated part of the equilibrium formula (1.5) with the simultaneous generation of spatial statistical links $(g \neq 0)$. The successive taking into account of spatial correlations in the hydrodynamic limit is of fundamental importance in the investigation of gas stream structure near the stability threshold /8,9/, and is a feature of the present investigation which distinguishes it from known investigations /5,6,11/ in the kinetic theory of hydrodynamic fluctuations.

Let us pass to the hydrodynamic description of transport and fluctuation processes in a nonequilibrium gas. From (1.1) we obtain the system of equations of transport for mean values of hydrodynamic fields /12/, whose abbreviated form

$$\frac{\partial}{\partial t}\overline{\Phi}_\alpha(t, \mathbf{r}) + \Theta_\alpha(\overline{\Phi}; \mathbf{r}) = H_\alpha(\overline{\Phi}; \mathbf{r}), \qquad \alpha = 0, 1, 2, 3, 4 \tag{1.7}$$

is used subsequently. We denote by $\Theta_{\alpha\beta}'$ and $H_{\alpha\beta}'$ the linearized operators

$$\Theta_{\alpha\beta}'(\overline{\Phi}; \mathbf{r})\, z(\mathbf{r}) = (z, \partial_{\overline{\Phi}_\beta})\, \Theta_\alpha(\overline{\Phi}; \mathbf{r}),\ H_{\alpha\beta}'(\overline{\Phi}; \mathbf{r})\, z(\mathbf{r}) = (z, \partial_{\overline{\Phi}_\beta})\, H_\alpha(\overline{\Phi}; \mathbf{r})$$

where

$$(z, \partial_{\overline{\Phi}_\beta}) = \int d\mathbf{r}' z(\mathbf{r}')\, \delta/\delta\overline{\Phi}_\beta(\mathbf{r}'), \quad H_0 = 0,\ H_k = -(m\bar{n})^{-1}\nabla_l \bar{P}_{lk}, \quad k = 1, 2, 3, \quad H_4 = -n^{-1}(\nabla_l \bar{q}_l + \bar{P}_{kl}\nabla_k \bar{u}_l)$$

$\delta/\delta\overline{\Phi}_\beta(\mathbf{r})$ is the functional derivative, $\overline{\Phi} = (\overline{\Phi}_0, \overline{\Phi}_1, \overline{\Phi}_2, \overline{\Phi}_3, \overline{\Phi}_4) = (\bar{n}, \bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{e})$ are mean values of density n, of hydrodynamic velocity $u_k\ (k = 1, 2, 3)$ and of thermal energy $e = 3kT/2$, $\Theta_\alpha(\overline{\Phi})$ is the nonlinear Euler operator, and $\bar{q}_l$ and $\bar{P}_{kl}$ are mean values of the heat flux and of viscous stress deviator. Here and below recurrent Latin subscripts indicate summation from 1 to 3, and the Greek ones summation from 0 to 4.

Using the system of additive collision invariants

$$\psi_0 = 1,\ \psi_k = \bar{n}^{-1}c_k \quad (k = 1, 2, 3),\ \psi_4 = \bar{n}^{-1}(mc^2 - \bar{e}), \quad c_k = v_k - \bar{u}_k$$

we represent fluctuations of hydrodynamic fields $\delta\Phi_0 = \delta n$, $\delta\Phi_k = \delta u_k$, $\delta\Phi_4 = \delta e$ in the form

$$\delta\Phi_\alpha(t, \mathbf{r}) = \int d\mathbf{v}\, \psi_\alpha\, \delta N(t, \mathbf{r}, \mathbf{v}), \quad \alpha = 0, 1, 2, 3, 4 \tag{1.8}$$

Formula (1.3) implies that the realization of the random field $\delta I(t, x)$ belongs to phase function subspace which is orthogonal to subspace spanning the collision invariants, i.e.

$$\int dv \psi_\alpha \delta I(t, \mathbf{r}, \mathbf{v}) = 0; \ \alpha = 0, 1, 2, 3, 4 \tag{1.9}$$

hence the calculation of moments of Eq. (1.2) with allowance for (1.8) yields for the hydrodynamic functions the system of transport equations

$$\frac{\partial}{\partial t}\delta n + \Theta'_{0\beta}(\overline{\Phi})\delta\Phi_\beta = 0, \quad \frac{\partial}{\partial t}\delta u_k + \Theta'_{k\beta}(\overline{\Phi})\delta\Phi_\beta = -\delta\left[\frac{1}{m\bar{n}}\nabla_l P_{lk}\right] \tag{1.10}$$

$$\frac{\partial}{\partial t}\delta e + \Theta'_{4\beta}(\overline{\Phi})\delta\Phi_\beta = \frac{1}{\bar{n}}\nabla_k(\bar{p}\delta_{kl} + \bar{P}_{kl})\delta u_l - \delta\left[\frac{1}{\bar{n}}\nabla_k q_k + \frac{1}{n}P_{kl}\nabla_l u_k\right]$$

where δ denotes a linear variation of the quantities appearing in square brackets, for example, $\delta[n^{-1}P_{kl}\nabla_k u_l] = -\bar{n}^2\delta n\bar{P}_{kl}\nabla_l u_k + \bar{n}^{-1}\delta P_{kl}\nabla_l u_k + \bar{n}^{-1}\bar{P}_{kl}\nabla_l\delta u_k$; the mean value of heat flux $\bar{q}_k$, of the stress tensor $\bar{P}_{kl}$, and of their fluctuations δq_k and δP_{kl} are defined by the formulas

$$\bar{q}_k(t, \mathbf{r}) = \frac{m}{2}\int d\mathbf{v} c_k c^2 F(t, x), \quad \bar{P}_{kl}(t, \mathbf{r}) = m\int d\mathbf{v}(c_k c_l)_s F(t, x) \tag{1.11}$$

$$\delta q_k(t, \mathbf{r}) = \frac{m}{2}\int d\mathbf{v} c_k c^2 \delta N(t, x), \quad \delta P_{kl}(t, \mathbf{r}) = m\int d\mathbf{v}(c_k c_l)_s \delta N(t, x)$$

where $(c_k c_l)_s = 2^{-1}(c_k c_l + c_l c_k) - 3^{-1}\delta_{kl}c^2$.

To close the transfer equations (1.10) and (1.7) it is necessary to determine the functional dependence of $\bar{q}_k$ and $\bar{P}_{kl}$ on $\overline{\Phi}$, and δq_k and δP_{kl} on $\overline{\Phi}$ and $\delta\Phi$. As shown in /12/, the first of these problems is solved by constructing standard solutions of Eq. (1.1). The Chapman—Enskog method yields $\bar{q}_k = -\bar{\lambda}\nabla_k\bar{T}$, $\bar{P}_{kl} = -2\bar{\eta}(\nabla_k\bar{u}_l)_s$, where $\bar{\lambda} = \lambda(\bar{T})$ and $\bar{\eta} = \eta(\bar{T})$ are the coefficients of thermal conductivity and viscosity, that are the Fourier and Newton's laws, with an accuracy to terms of the order of K (the Knudsen number). To utilize this result it is necessary to modify in Eqs. (1.10) the Chapman—Enskog method in its application to the stochastic kinetic equation (1.2), to derive its standard solution with the same degree of accuracy (of the order of K), and calculate in that approximation fluctuations of thermodynamic fluxes, using formulas (1.11). The execution of this scheme is given below. Its basic result is the derivation of formulas

$$\delta q_k(t, \mathbf{r}) = (\bar{p}\delta_{kl} + \bar{P}_{kl})\delta u_l - \delta[\lambda(T)\nabla_k T] + \delta Q_k(t, \mathbf{r}) \tag{1.12}$$

$$\delta P_{kl}(t, \mathbf{r}) = -\delta[2\eta(T)(\nabla_k u_l)_s] + \delta\Pi_{kl}(t, \mathbf{r})$$

where δQ_k and $\delta\Pi_{kl}$ represent external sources of the heat flux and stress tensor fluctuations, which are random Gaussian fields with zero mean value and the correlation functions

$$\langle\delta Q_k(1)\delta Q_l(2)\rangle = 2k\bar{T}^2\bar{\lambda}\delta(1-2)\left[\delta_{kl} + \frac{9}{20}\frac{\bar{P}_{kl}}{\bar{p}}\right] \tag{1.13}$$

$$\langle\delta\Pi_{kl}(1)\delta\Pi_{sp}(2)\rangle = 4k\bar{T}\bar{\eta}\delta(1-2)E^{kl}_{in}E^{sp}_{nt}\left[\delta_{it} + \frac{\bar{P}_{it}}{\bar{p}}\right]$$

$$\langle\delta Q_s(1)\delta\Pi_{kl}(2)\rangle = \frac{27}{5\bar{n}}\bar{\eta}\delta(1-2)E^{kl}_{sp}\bar{q}_p$$

where the arguments 1 and 2 of functions denote the sets $(t_1, \mathbf{r}_1)$ and $(t_2, \mathbf{r}_2)$, and $\bar{p}$ is the hydrostatic pressure

$$E^{kl}_{sp} = \frac{1}{2}(\delta_{pk}\delta_{sl} + \delta_{pl}\delta_{sk}) - \frac{1}{3}\delta_{kl}\delta_{sp}$$

Taking into account formulas (1.12) and the Fourier and Newton's laws for $\bar{q}_k$ and $\bar{P}_{kl}$, we transform Eqs. (1.10) into the linearized Navier—Stokes—Fourier equations

$$\frac{\partial}{\partial t}\delta\Phi_\alpha(t, \mathbf{r}) = [H'_{\alpha\beta} - \Theta'_{\alpha\beta}]\delta\Phi_\beta(t, \mathbf{r}) + \delta G_\alpha(t, \mathbf{r}), \quad \alpha = 0, 1, 2, 3, 4 \tag{1.14}$$

with the random external sources δG

$$\delta G_0 = 0, \ \delta G_k = -\bar{n}^{-1}\nabla_l\delta\Pi_{lk}, \ k = 1, 2, 3, \ \delta G_4 = -\bar{n}^{-1}\nabla_k\delta Q_k - \bar{n}^{-1}\delta\Pi_{kl}\nabla_l\bar{u}_k$$

For a homogeneous gas ($\bar{q} = 0, \bar{P} = 0$) formulas (1.13) become the known expressions of the fluctuation-dissipation theorem for the correlators of equilibrium fluctuations of thermodynamic fluxes, which were first obtained on phenomenological grounds /7/ and, also, in the kinetic theory for the thermodynamical equilibrium state /13/ and for the local equilibrium state /5/.

The new result of this investigation is the determination of nonequilibrium additions $\bar{P}_{kl}/\bar{p}$ in the first two formulas of (1.13) and of the last formula (1.13). These formulas are generalizations of relations for the hydrodynamic evolution stage of nonhomogeneous gas. Note that they do not contain physical parameters other than $\tilde{\lambda}$ and $\tilde{\eta}$.

From Eqs.(1.14) and formulas (1.13) we can obtain the representation of the simultaneous two-point moment of the hydrodynamic field fluctuations $\langle\delta\Phi_\alpha(t, \mathbf{r}_1)\,\delta\Phi_\beta(t, \mathbf{r}_2)\rangle$ in the form of the sum of δ-correlation term $b_{\alpha\beta}^{(0)}(t, \mathbf{r}_1, \mathbf{r}_2)$ and of the space correlator $b_{\alpha\beta}^{(1)}(t, \mathbf{r}_1, \mathbf{r}_2)$ which is conformity with (1.5) are defined by formulas

$$b_{\alpha\beta}^{(0)} = \delta(\mathbf{r}_1 - \mathbf{r}_2)\int d\mathbf{v}_1\psi_\alpha(\mathbf{v}_1)\,\psi_\beta(\mathbf{v}_1)\,F(t, x_1), \quad b_{\alpha\beta}^{(1)} = \int d\mathbf{v}_1\,d\mathbf{v}_2\psi_\alpha(\mathbf{v}_1)\,\psi_\beta(\mathbf{v}_2)\,g \tag{1.15}$$

(F and g are the standard solutions of Eqs.(1.1) and (2.6), respectively), and, also, to derive a closed system of hydrodynamic equations for $b_{\alpha\beta}^{(1)}$. The derivation of these equations is fairly cumbersome and is omitted here. It is, however, important to note that the nonequilibrium additions to $b_{\alpha\beta}^{(0)}$ in the form of F_1 and F_0 that correspond to locally equilibrium distribution in (1.15) prove to be quantities of the same order as the nonequilibrium correlator $b_{\alpha\beta}^{(1)}$ (for a homogeneous gas $b_{\alpha\beta}^{(1)} = 0$). It is, thus, necessary, when extending the Onsager method of calculating hydrodynamic fluctuations to the nonequilibrium domain of initial conditions to the linearized Navier—Stokes—Fourier equations for the two-transient moments of random fields $\delta\Phi_\alpha(t, \mathbf{r})$ must be specified in the form of the sum $b_{\alpha\beta}^{(0)} + b_{\alpha\beta}^{(1)}$ where the spatial correlators $b_{\alpha\beta}^{(1)}$ are determined by formulas obtained beforehand as the result of solving respective inhomogeneous equations.

The difference of the obtained here results from those in /5,6,11/ which dealt with the kinetic theory of unbalanced hydrodynamic fluctuations should be noted. For instance, only equilibrium terms were obtained in /5,6/ in formulas (1.13) (the Landau—Lifshitz formulas). The erroneous conclusions in these investigations are due to several causes. In /5/ the calculation of correlator $\langle\delta I\delta I\rangle$ formula (1.3) did not contain the term $J_{v_1v_2}$ without which it contradicts the laws of conservation. Owing to this the author of /5/ had to limit the analysis of formula (1.5) to the case of $F = F_0$, i.e. to neglect, in fact, all effects of unbalance. In /6/ the projection method, whose application is apparently limited to the domain of small deviation from equilibrium, was used for the derivation of hydrodynamic equations. In the extension of the Onsager method to the domain of nonequilibrium states the δ-correlation of simultaneous two-point moments of fluctuations of hydrodynamic fields was, in fact, postulated in /11/. Because in that paper spacial statistical links ($b_{\alpha\beta}^{(1)} = 0$) were not taken into account, the conclusion was made about the fluctuation intensity level being normal near the hydrodynamic stability threshold, which contradicts experimental data and the result presented in /5/. Actually at the stability threshold it is the behavior of spatial correlators $b_{\alpha\beta}^{(1)}$ that is anomalous.

2. Solution of the stochastic equation (1.2) using the Chapman—Enskog method. The class of standard solutions of the gaskinetic Boltzmann equation (1.1) by the Chapman—Enskog method is asymptotic in the domain of low Knudsen numbers $K = l/L$, where l is the /mean/ free path length and L is a characteristic macroscopic length scale. Application of this method to the solution of the stochastic kinetic equation (1.2) necessitates a preliminary analysis of the order of magnitude of its individual terms in the space-time scales that are characteristic of the hydrodynamic stage of gas evolution. The estimate of the magnitude of phase density fluctuation δN relative to the mean value is essential. In the linear theory δN is assumed small in comparison with F. A more rigorous estimate for thermodynamic equilibrium state $(g = 0)$: $\delta N \sim \sqrt{v^\circ/L^3}F_0$, where $v^\circ = V/N^\circ$ and V is the system volume, is obtained from (1.5). There are no physical grounds to assume any significant difference of fluctuation intensity in various states on a continuous thermodynamic branch. Hence the relation $\delta N \sim \sqrt{v^\circ/L^3}F$ is used below also for nonequilibrium but stable states. This relation is only violated in the narrow region close to the stability threshold.

The introduction of dimensionless variables $t' = t/\tau$, $\mathbf{r}' = \mathbf{r}/L$, $\mathbf{v}' = \mathbf{v}/w$, where w is the thermal speed and $\tau = L/w$, and of dimensionless functions $F' = v^\circ w^3 F$ and $\delta N' = w^3\sqrt{L^3v^\circ}\delta N$ leads to the appearance in (1.2) of parameter K, and it is necessary to take into account the estimate $\delta I' = w^3\tau\sqrt{L^3v^\circ}\delta I \sim K^{-1/2}$ which directly follows from (1.3). The formal introduction of parameter K in Eqs.(1.1) and (1.2) in conformity with the dimensionless variables for fixing the order of magnitude of individual terms requires the following formulas:

$$K\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)F = J_v(F, F) \tag{2.1}$$

$$K\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\delta N = J_v'(F)\,\delta N + K^{1/2}\delta I(t, x; K) \tag{2.2}$$

where the dependence of δI on K is, by virtue of (1.3), determined by the corresponding dependence of F on K. In final formulas K is set equal to unity.

The asymptotic expansion $F = F_0 + KF_1 + K^2F_2 + \ldots$ of solution of Eq.(2.1) leads to the corresponding expansion of the collision operator $J_v'(F) = J_v'(F_0) + KJ_v'(F_1) + \ldots$ and of the fluctuating collision integral $\delta I = \delta I_0 + K^{1/2}\delta I_1 + K\delta I_2 + \ldots$, where $\delta I_0, \delta I_1, \ldots$ are statistically independent Gaussian fields with zero mean value and correlation functions, respectively, determined by the first, second, etc. terms in the expansion (1.3) in series in integral powers of K. The asymptotic behavior of solutions of Eq.(2.2) for $K \ll 1$ is consistent with the formal expansion of the form

$$\delta N = \delta N_0 + K^{1/2}\delta N_1 + K\delta N_2 + K^{3/2}\delta N_3 + \ldots \tag{2.3}$$

The coefficients of series (2.3) can be determined by the standard Chapman—Enskog method /12/ according to which the formal expansion (2.3) generates the operator expansion $\partial/\partial t = \partial_0/\partial t + K^{1/2}\partial_1/\partial t + K\partial_2/\partial t + \ldots$. The method of determining $\partial_n/\partial t$ is based on the use of indecomposability of $\delta\Phi_\alpha$

$$\int \partial \mathbf{v}\psi_\alpha \delta N_n = \delta_{0n}\delta\Phi_\alpha, \quad \alpha = 0,\ 1,\ 2,\ 3,\ 4 \tag{2.4}$$

For the first M terms of series (2.3) this method yields the expressions

$$q_k = \sum_{n=0}^{M} \delta q_k^{(n)} \equiv \sum_{n=0}^{M} \frac{m}{2}\int d\mathbf{v} c_k c^2 \delta N_n, \quad \delta P_{kl} = \sum_{n=0}^{M} \delta P_{kl}^{(n)} \equiv \sum_{n=0}^{M} m \int d\mathbf{v}\,(c_k c_l)_s\, \delta N_n \tag{2.5}$$

for the thermodynamic flux fluctuations in Eqs.(1.10).

The following formulas:

$$F_0 = \bar{n}\,(3m/4\,\pi\bar{e})^{3/2} \exp\,[-\,3\,mc^2/(4\bar{e})], \quad F_1 = F_0\,h \equiv -\,F_0\bar{n}^{-1}\,[A_k\nabla_k \ln \bar{T} + B_{kl}\nabla_l \bar{u}_k]$$

can be used for F_0 and F_1 /12/. In these formulas $A_k(\mathbf{c})$ and $B_{kl}(\mathbf{c})$ are defined by the equations

$$\bar{n}L_v A_k = F_0\,[mc^2/(2k\bar{T}) - {}^5/_2]\,c_k, \quad \bar{n}L_v B_{kl} = F_0 m\,(c_k c_l)_s\,/\,(k\bar{T}), \quad L_v = -\,\bar{n}^{-2}F_0 J_v'(F_0)$$

Taking into account these formulas, for the first three terms of series (2.3) we obtain

$$J_v'(F_0)\,\delta N_0 = 0, \quad J_v'(F_0)\,\delta N_1 = -\,\delta I_0, \quad J_v'(F_0)\,\delta N_2 = (\partial_0/\partial t + \mathbf{v}\cdot\nabla)\,\delta N_0 - J_v'(F_1)\,\delta N_0 - \delta I, \tag{2.6}$$

$$\langle\delta I_0\rangle = \langle\delta I_1\rangle = \langle\delta I_0 \delta I_1\rangle = 0 \tag{2.7}$$

$$\begin{aligned}\langle\delta I_0(1,\mathbf{v}_1)\,\delta I_0(2,\ \mathbf{v}_2)\rangle &= -\,\delta\,(1{-}2)\,[J_{v_1}'(F_0) + \\ &J_{v_2}'(F_0)]F_0\delta\,(\mathbf{v}_1 - \mathbf{v}_2)\ \langle\delta I_1(1,\ \mathbf{v}_1)\,\delta I_1(2,\ \mathbf{v}_2)\rangle = \\ &\delta\,(1{-}2)\,\{\delta\,(\mathbf{v}_1 - \mathbf{v}_2)\,J_{v_1}'(F_0)\,F_1 + J_{v_1v_2}(F_0,\,F_1) + \\ &J_{v_1v_2}(F_1,\,F_0) - [J_{v_1}'(F_0) + J_{v_2}'(F_0)]\,F_1\delta\,(\mathbf{v}_1 - \mathbf{v}_2) + \\ &[\,J_{v_1}'(F_1) + J_{v_2}'(F_1)]\,F_0\,\delta\,(\mathbf{v}_1 - \mathbf{v}_2)\}\end{aligned} \tag{2.8}$$

Using formulas (2.8) it is possible to prove that the random fields δI_0 and δI_1 posess properties (1.9).

Solution of first two of Eqs.(2.6) with allowance for (2.4) yields

$$\delta N_0 = F_0 \sum_{\alpha=0}^{4} \gamma_\alpha^{-1}\psi_\alpha\delta\Phi_\alpha = \partial\,(\delta\Phi)\,F_0(\bar{\Phi}), \quad \gamma_\alpha = \int d\mathbf{v}\psi_\alpha^2 F_0 \tag{2.9}$$

$$\delta N_1 = \bar{n}^{-2}\,F_0 L_v^{-1}\,\delta I_0\,(t,\,x) \tag{2.10}$$

The explicit form of operator $\partial = \partial\,(\delta\Phi)$ is of the form $\partial = (\delta\,\Phi, \partial_{\bar{\Phi}_\alpha})$. Its action on the product of the hydraulic field mean values is similar to that of the introduced above operator of the linear variation δ on the product of random hydrodynamic fields.

Before proceeding with the solution of the third of Eqs.(2.6) it is necessary to determine operator $\partial_0/\partial t$ using equations

$$\int d\mathbf{v}\psi_\alpha\,[\partial_0/\partial t + \mathbf{v}\cdot\nabla]\,\partial\,(\delta\Phi)\,F_0(\bar{\Phi};\,\mathbf{v}) = 0, \quad \alpha = 0, 1, 2, 3, 4 \tag{2.11}$$

which with allowance for (2.4), (2.9), and (1.9), follow from Eq.(2.6) for δN_2.

The integrand in (2.11) can be transformed into (see the Appendix)

$$[\partial_0/\partial t + \mathbf{v}\cdot\nabla]\,\delta N_0 = \partial\,(\delta\Phi)\,J_v'(F_0)\,F_1 + (\partial_0/\partial t\,\delta\Phi_\alpha + \Theta_{\alpha\beta}'\delta\Phi_\beta)\,F_0\gamma_\alpha^{-1}\psi_\alpha \tag{2.12}$$

It is obvious that the substitution of this expression into (2.11) results is the cancellation of the first term in its right-hand side owing to the orthogonality of $J_v'(F_0)F_1$ to the subspace stretched over the collision invariants, and that the remaining terms yield the relationship $\partial_0\delta\Phi_\alpha/\partial t = -\Theta'_{\alpha\beta}\delta\Phi_\beta$.

Using the properties of the linearized collision operator $J_v'(F)G_1 = J_v'(G)F$ and taking into account the obtained expression for $\partial_0\delta\Phi_\alpha/\partial t$, we obtain

$$[\partial_0/\partial t + \mathbf{v}\cdot\nabla]\,\delta N_0 = \partial(\delta\Phi)J_v'(F_0)F_1 = J_v'(F_1)\,\delta N_0 + J_v'(F_0)\,\partial(\delta\Phi)\,F_1$$

whose substitution into the right-hand side of Eq. (2.6) for δN_2 reduces it to the form

$$J_v'(F_0)\,[\delta N_2 - \partial(\delta\Phi)\,F_1] = -\,\delta I_1.$$

Its obvious solution is

$$\delta N_2(t,\ x) = \partial(\delta\Phi)\,F_1(t,x) + \bar{n}^{-2}F_0\,L_v^{-1}\delta I_1(t,\ x) \qquad (2.13)$$

The substitution of derived solutions (2.9), (2.10), and (2.13) into formulas (2.5) yields, after the determination of integrals of the velocity space, the following formulas:

$$\delta q_k^{(0)} = \bar{p}\delta u_k,\quad \delta P_{kl}^{(0)} = 0,\quad \delta q_k^{(1)} = \delta Q_k^{(0)},\quad \delta P_{kl}^{(1)} = \delta\Pi_{kl}^{(0)} \qquad (2.14)$$

$$\delta q_k^{(2)} = \partial(\delta\Phi)\,\bar{q}_k(\overline{\Phi}) + \bar{P}_{kl}\delta u_l + \delta Q_k^{(1)}$$

$$\delta P_{kl}^{(2)} = \partial(\delta\Phi)\,\bar{P}_{kl}(\overline{\Phi}) + \delta\Pi_{kl}^{(1)}$$

$$\delta Q_k^{(i)} = \frac{m}{2\bar{n}^2}\int d\mathbf{v}\,c_k c^2 F_0 L_v^{-1}\delta I_i = \frac{kT}{\bar{n}}\int d\mathbf{v}\,A_k\delta I_i \qquad (2.15)$$

$$\delta\Pi_{kl}^{(i)} = \frac{m}{\bar{n}^2}\int d\mathbf{v}\,(c_k c_l)_s\,F_0 L_v^{-1}\delta I_i = \frac{kT}{\bar{n}}\int d\mathbf{v}\,B_{kl}\delta I_i,\quad i = 0, 1$$

The previously given determination of functions $A_k(\mathbf{c})$ and $B_{kl}(\mathbf{c})$ is used in (2.15) and the self-conjugacy of operator L_v is taken into account. Introducing the notation $\delta Q_k = \delta Q_k^{(0)} + \delta Q_k^{(1)}$ and $\delta\Pi_{kl} = \delta\Pi_{kl}^{(0)} + \delta\Pi_{kl}^{(1)}$ and taking into account (2.14) from formulas (2.5) we obtain expressions (1.12) when $M = 2$ and form the statistical properties of random fields δI_0 and δI_1, and formulas (2.15) we have the Gaussian properties and space-time δ-correlation of external sources of thermodynamic flux fluctuations δQ_k and $\delta\Pi_{kl}$.

Thus in the zero approximation with respect to K the hydrodynamic equations (1.10) for fluctuations are linearized Euler equations; expression $\delta\mathbf{q} = \bar{p}\delta\mathbf{u}$ for thermal flux fluctuations cancels with the respective term in Eq. (1.10) for δe. If the mean values of thermodynamic fluxes are taken into account in the first order with respect to K, their fluctuations must also be taken into account with the same degreee of accuracy $\delta\mathbf{q} = \delta\mathbf{q}^{(0)} + \delta\mathbf{q}^{(1)} + \delta\mathbf{q}^{(2)}$, $\delta\mathbf{P} = \delta\mathbf{P}^{(0)} + \delta\mathbf{P}^{(1)} + \delta\mathbf{P}^{(2)}$. The terms $\delta\mathbf{q}^{(1)}$ and $\delta\mathbf{P}^{(1)}$ of order $K^{1/2}$ are purely stochastic and, by virtue of (2.15), independent of $\bar{\mathbf{q}}$ and $\overline{\mathbf{P}}$. It is shown below that they define fluctuation sources in the locally balanced state, while the terms $\delta\mathbf{q}^{(2)}$ and $\delta\mathbf{P}^{(2)}$ take into account, as expected, the damping of fluctuations owing to viscosity and thermal conductivity, as well as their simultaneous generation by external unbalanced sources $\delta Q^{(1)}$ and $\delta\Pi^{(1)}$.

Note that in /3,7/ both terms in the right-hand side of Eq. (1.2) were assumed to be quantities of the same order K^{-1} which, as can be readily checked, corresponds to the false estimate $\delta N \sim F$ and results in an expansion of δN in integral powers of K. However the hydrodynamic equations for fluctuations are unaffected in the case of thermodynamic equilibrium state. For inhomogeneous states it manifests itself in the first approximation with respect to K in that the nonequilibrium fluctuation sources $\delta\mathbf{Q}^{(1)}$ and $\delta\mathbf{\Pi}^{(1)}$ remain unaccounted for.

3. Calculation of correlators of fluctuating thermodynamic fluxes.

We introduce the abbreviated notation $\mathbf{C}_1 = \mathbf{A}$, $\delta\mathbf{S}_1^{(i)} = \delta\mathbf{Q}^{(i)}$, $\mathbf{C}_2 = \mathbf{B}$, $\delta\mathbf{S}_2^{(i)} = \delta\mathbf{\Pi}^{(i)}$, $i = 0, 1$, where $\mathbf{C}_1$ and $\delta\mathbf{S}_1^{(i)}$ are vector functions, and $\mathbf{C}_2$ and $\delta\mathbf{S}_2^{(i)}$ are second rank tensors. The space-time correlators of random fields $\delta\mathbf{Q}^{(i)}$ and $\delta\mathbf{\Pi}^{(i)}$, $i = 0, 1$ can be expressed, using formulas (2.15), in terms of correlators of the fluctuating collision integrals δI_0 and δI_1 as follows:

$$\langle\delta\mathbf{S}_a^{(i)}(1)\,\delta\mathbf{S}_b^{(j)}(2)\rangle = \left(\frac{kT}{\bar{n}}\right)^2\int d\mathbf{v}_1 d\mathbf{v}_2\,\mathbf{C}_a(1,\ \mathbf{v}_1)\,\mathbf{C}_b(2,\ \mathbf{v}_2)\,\langle\delta I_i(1,\ \mathbf{v}_1)\,\delta I_j(2,\ \mathbf{v}_2)\rangle,\ i, j = 0,1;\ a, b = 1,2 \qquad (3.1)$$

It is then necessary to substitute into formulas (3.1) expressions (2.8) and calculate the integrals over the velocity space. Several technical problems are encountered in the process, in particular in connection with very unwieldy transformations of integrands for $i = 1$. Another, shorter path makes possible the expression of correlators (3.1) directly in terms of standard integral brackets (or Ω integrals). For this the expression for function $D\{F, F; \mathbf{v}_1,$

$\mathbf{v}_2]$ in (1.3) must be represented in the form

$$D[F, F;\ \mathbf{v}_1, \mathbf{v}_2] = \frac{1}{2}\int d\xi_1 d\xi_2 d\xi_1' d\xi_2' \sigma(\xi_1\xi_2 \to \xi_1'\xi_2') F(\xi_1) F(\xi_2) \varkappa(\xi_1\xi_2\xi_1'\xi_2';\ \mathbf{v}_1) \times \varkappa(\xi_1\xi_2\xi_1'\xi_2';\ \mathbf{v}_2) \qquad (3.2)$$

It should be pointed out that the standard definition of the integral bracket $[W; G]$ /12/ for two arbitrary functions $W(\mathbf{v})$ and $G(\mathbf{v})$ can be transformed, using (3.2) into

$$[\bar{W}; G] = \int d\mathbf{v} \bar{W}(\mathbf{v}) L_v G(\mathbf{v}) = (2\bar{n}^2)^{-1} \int d\mathbf{v}_1 d\mathbf{v}_2 W(\mathbf{v}_1) G(\mathbf{v}_2) D[F_0, F_0;\ \mathbf{v}_1, \mathbf{v}_2] \qquad (3.3)$$

$$\varkappa(\mathbf{v}_1\mathbf{v}_2\mathbf{v}_1'\mathbf{v}_2';\ \mathbf{v}) = \delta(\mathbf{v} - \mathbf{v}_1') + \delta(\mathbf{v} - \mathbf{v}_2') - \delta(\mathbf{v} - \mathbf{v}_1) - \delta(\mathbf{v} - \mathbf{v}_2)$$

where $\sigma(\mathbf{v}_1\mathbf{v}_2 \to \mathbf{v}_1'\mathbf{v}_2')$ is the dissipation cross section. The proof that formulas (1.3) and (3.2) are identical can be found in /3/, and the validity of formula (3.3) can be directly verified.

Taking into account formula (3.2) for D, formula (2.8) can be written as

$$\langle \delta I_i(1, \mathbf{v}_1)\, \delta I_j(2, \mathbf{v}_2)\rangle = \delta_{ij}\, \delta(1-2)\, D^{(i)}(\mathbf{v}_1 - \mathbf{v}_2),\ i, j = 0, 1 \qquad (3.4)$$

$$D^{(0)}(\mathbf{v}_1, \mathbf{v}_2) = D[F_0, F_0;\ \mathbf{v}_1, \mathbf{v}_2],\quad D^{(1)}(\mathbf{v}_1, \mathbf{v}_2) = \frac{1}{2} D[F_0, F_1;\ \mathbf{v}_1, \mathbf{v}_2]$$

whose substitution into (3.1) using the definition (3.3) of the integral bracket allows us to write

$$\langle \delta S_a^{(i)}(1)\, \delta S_b^{(j)} \bar{T}(2)\rangle = 2(k\bar{T})^2 \delta_{ij} \delta(1-2)\, \Lambda^{(i)}(\mathbf{C}_a, \mathbf{C}_b) \qquad (3.5)$$

$$\Lambda^{(0)}(\mathbf{C}_a, \mathbf{C}_b) = [\mathbf{C}_a;\ \mathbf{C}_b] \qquad (3.6)$$

$$2\Lambda^{(1)}(\mathbf{C}_a, \mathbf{C}_b) = [\mathbf{C}_a;\ h\mathbf{C}_b] + [\mathbf{C}_b;\ h\mathbf{C}_a] - [h;\ \mathbf{C}_a\mathbf{C}_b] + [\mathbf{C}_a;\ h\mathbf{C}_b]^* + [\mathbf{C}_b;\ h\mathbf{C}_a]^* - [h;\ \mathbf{C}_a\mathbf{C}_b]^*,\quad i, j = 0, 1,\quad a, b = 1, 2 \qquad (3.7)$$

where $[\cdot;\ \cdot]^*$ is a modified integral bracket which for the three arbitrary functions $R(\mathbf{v})$, $H(\mathbf{v})$ and $G(\mathbf{v})$ is determined by formula

$$[R;\ HG]^* = \frac{1}{\bar{n}^2}\int d\mathbf{v} R(\mathbf{v}) [J_v(F_0 H, F_0 G) + J_v(F_0 G, F_0 H)] \qquad (3.8)$$

Formula (3.6) directly follows from (3.1) and the definition (3.3) of the standard integral bracket. Formula (3.7) is obtained substituting (3.4) and (3.2) into (3.1) and integrating with respect to $\mathbf{v}_1$ and $\mathbf{v}_2$. Then, taking into account the symmetry of σ we separate in the obtained integral expression the terms that can be grouped in the standard integral brackets with the remaining terms grouped in modified integral brackets. Formulas (3.5)—(3.7) enable us to use the calculations of integral brackets given in /12/.

The integral brackets in (3.6) define, in essence, the transport coefficients /12/

$$k[A_k;\ A_l] = \bar{\lambda}\delta_{kl},\quad k\bar{T}[B_{kl};\ B_{st}] = 2\bar{\eta}E_{st}^{kl},\quad [A_k;\ B_{st}] = 0$$

Formulas (3.5) for correlators of random fields $\delta\mathbf{Q}^{(0)}$ and $\delta\mathbf{\Pi}^{(0)}$ reduce to the Landau—Lifshitz formulas /9/ and correspond to the first terms in formulas (1.14) and (1.15). Correlators (3.5) of random fields $\delta\mathbf{Q}^{(1)}$, $\delta\mathbf{\Pi}^{(1)}$ depend by virtue of (3.7) on the mean values on parameters of thermodynamic fluxes and yield, consequently, nonequilibrium additions to the Landau—Lifshitz correlation formulas. The integral brackets that constitue expression (3.7) for $\Lambda^{(1)}$ appear in the Barratt approximation of the Chapman—Enskog method.

Since we are primarily interested in the qualitative aspects of the gas nonhomogeneity effects on hydrodynamic fluctuations, we shall use Maxwellian molecules in calculating $\Lambda^{(1)}$. For these molecules all modified integral brackets in (3.7) are zero; $[h;\ \mathbf{BB}] = [\mathbf{B};\ h\mathbf{B}]$, $2[h;\ \mathbf{AA}] = 3[\mathbf{A};\ h\mathbf{A}]$, $[h;\ \mathbf{AB}] = [\mathbf{B};\ hA]$, and the nontrivial contribution to (3.7) is provided by the brackets

$$[B_{kl};\ hB_{st}] = \frac{4}{\bar{n}}(k\bar{T})^{-2}\bar{\eta}\bar{P}_{ij}E_{in}^{kl}E_{nj}^{st} \qquad (3.9)$$

$$[A_k;\ hA_l] = \frac{9}{5\bar{n}k}(k\bar{T})^{-1}\bar{\lambda}\bar{P}_{kl}$$

$$[B_{kl};\ hA_s] = \frac{27}{5\bar{n}}(k\bar{T})^{-2}\bar{\eta}\bar{q}_p E_{ps}^{kl}$$

Substitution of these expressions into (3.7) results in that in (3.5) we obtain for $i = j = 1$ formulas that are the same as the second terms of the first two expressions in (1.13) and as the third of formulas (1.13). Since the expressions in (3.9) are bilinear with respect to transport coefficients, the terms corresponding to them in (1.13) represent the squared response of the system to thermal perturbations in inhomogeneous stable states of gas.

Formulas (3.5)—(3.7) for the correlators of external sources of thermodynamic flux fluctuations are most comprehensive and are valid for any intermolecular interaction potential in a simple gas, and are compatible with the condition of existence of the collision integral. The formulas in (1.13) represent the estimate for (3.5) that corresponds to the first approximation in the expansion of integral brackets in (3.7) in Sonin's polynomials (Maxwellian gas). Note that the general form of dependence of correlators of external thermodynamic fluctuation sources on mean values of thermodynamic fluxes, specified by formulas (1.13) applied to all subsequent approximations. Hence estimate (1.13) for (3.5) is evidently reasonably accurate.

Appendix. The cumbersome computations involved in the direct proof of formula (2.12) can be avoided by using the expression $\partial_0/\partial t = (\partial_0\overline{\Phi}_\alpha/\partial t,\ \partial_{\overline{\Phi}_\alpha}) + (\partial_0\delta\Phi_\alpha/\partial t, \partial_{\delta\Phi_\alpha})$ which is directly implied by the definition of operator $\partial_0/\partial t$:

$$\left[\frac{\partial_0}{\partial t} + \mathbf{v}\cdot\nabla\right]\delta N_0 = (\partial_0\overline{\Phi}_\alpha/\partial t, \partial_{\overline{\Phi}_\alpha})\,\delta N_0 + \mathbf{v}\cdot\nabla\delta N_0 + (\partial_0\delta\Phi_\alpha/\partial t, \partial_{\delta\Phi_\alpha})\,\delta N_0$$

With allowance for (2.9) we carry out differentiation in the second and third terms and obtain

$$\mathbf{v}\cdot\nabla\delta N_0 = \partial(\delta\Phi)\,\mathbf{v}\cdot\nabla F_0, \quad (\partial_0\delta\Phi_\alpha/\partial t, \partial_{\delta\Phi_\alpha})\,\delta N_0 = (\partial_0\delta\Phi_\alpha/\partial t, \partial_{\overline{\Phi}_\alpha})\,F_0$$

Using the relations

$$\partial(\delta\Phi)(\partial_0\overline{\Phi}_\alpha/\partial t, \partial_{\overline{\Phi}_\alpha})\,F_0 = -\partial(\delta\Phi)(\Theta_\alpha, \partial_{\overline{\Phi}_\alpha})\,F_0 = -(\Theta'_{\alpha\beta}\delta\Phi_\beta, \partial_{\overline{\Phi}_\alpha})\,F_0 - (\Theta_\alpha, \partial_{\overline{\Phi}_\alpha})\ \partial(\delta\Phi)\,F_0 = -(\Theta'_{\alpha\beta}\delta\Phi_\beta, \partial_{\overline{\Phi}_\alpha})\,F_0 + (\partial_0\overline{\Phi}_\alpha/\partial t, \partial_{\overline{\Phi}_\alpha})\,\delta N_0$$

in which the equalities $\partial_0\overline{\Phi}_\alpha/\partial t = -\Theta_\alpha[\Phi]$ and $\partial(\delta\Phi)\,\Theta_\alpha[\overline{\Phi}] = \Theta'_{\alpha\beta}\delta\Phi_\beta$ are taken into account we reduce the first term to the form

$$(\partial_0\overline{\Phi}_\alpha/\partial t, \partial_{\overline{\Phi}_\alpha})\,\delta N_0 = \partial(\delta\Phi)\frac{\partial_0}{\partial t}F_0 + (\Theta'_{\alpha\beta}\delta\Phi_\beta, \partial_{\overline{\Phi}_\alpha})\,F_0$$

with

$$\left[\frac{\partial_0}{\partial t} + \mathbf{v}\cdot\nabla\right]\delta N_0 = \partial(\delta\Phi)\left[\frac{\partial_0}{\partial t} + \mathbf{v}\cdot\nabla\right]F_0 + \left(\left[\delta_{\alpha\beta}\frac{\partial_0}{\partial t}\delta\Phi_\beta + \Theta'_{\alpha\beta}\delta\Phi_\beta\right], \partial_{\overline{\Phi}_\alpha}\right)F_0$$

Using here the equation $[\partial_0/\partial t + \mathbf{v}\cdot\nabla]\,F_0 = J_v'(F_0)\,F_1$ which respresents the first approximation of the Boltzmann equation with respect to the Knudsen parameter, we obtain relation (2.12).

REFERENCES

1. DE GROOT S. R. and MAZUR P., Nonequilibrium Thermodynamics. Moscow, MIR, 1964.
2. TOLUBINSKII E.V., Theory of Transport Processes. Kiev, NAUKOVA DUMKA, 1969.
3. KLIMONTOVICH Iu.L., Kinetic Theory of Imperfect Gas and Imperfect Plasma. Moscow, NAUKA, 1975.
4. GRECHANNYI O.A., The stochastic approach and functional models in the kinetic theory of gas. Teoret. i Matem. Fiz., Vol.36. No.2, 1978.
5. HINTON F.L., Nonequilibrium theory of fluid fluctuations. Phys. Fluid, Vol.13, No.4, 1970.
6. MASHIYAMA KAZUKO T. and MORI HAZIME, Origin of the Landau—Lifshitz hydrodynamic fluctuations in nonequilibrium systems and new method for reducing the Boltzmann equation. J. Statist. Phys., Vol.18, No.4, 1978.
7. LANDAU L.D. and LIFSHITZ E.M., On hydrodynamic fluctuations. English translation, Pergamon Press, Landau & Lifshitz, Theoretical Physics, Vol.6. Fluid Mechanics, 1959.
8. ZAITSEV V.M. and SHLIOMIS M.I., Hydrodynamic fluctuations near the convection threshold. ZhETF, Vol.59, No.5,1970.
9. KHON'KIN A.D., On the effect of nonequilibrium fluctuations on turbulence onset. In: Turbulent Flows. Moscow, NAUKA, 1974.

10. GRECHANNYI O.A., On the theory of small fluctuations in nonequilibrium gas. IN: Physics and Technology of Heat. Issue 30. Kiev, NAUKOVA DUMKA, 1979.

11. FISHER I. Z. and LESNIKOV V.P., Nonequilibrium thermal hydrodynamic fluctuations. In: Physics of the Fluid State. Issue 4. Kiev, VISZCZA SZKOLA, 1976.

12. CHAPMAN S. and COWLING T.G., The Mathematical Theory of Inhomogeneous Gases, Cambridge Univ. Press, 1960.

13. FOX R.F. and UHLENBECK G.E., Contributions to nonequilibrium thermodynamics, II. Fluctuation theory for the Boltzmann equation. Phys. Fluid, Vol.13, No.12, 1970.

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